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Gauge Invariance and the Electromagnetic Current of Composite Pions

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The Global Color-symmetry Model of QCD is extended to deal with a background electromagnetic field and the associated conserved current is identified for composite $\bar{q}q$ pion modes of the model. Although the analysis is limited to tree level in the bilocal fields that bosonize the model, the identified photon-pion vertex produces the charge form factor associated with ladder Bethe-Salpeter pion amplitudes. A Ward-Takahashi identity for this vertex is derived in terms of the effective inverse propagator for the equivalent local pion field and the intrinsic ladder Bethe-Salpeter amplitudes. This identity is then used to illustrate gauge invariance by showing that identical vertex information is produced from the gauge change of the free action once proper account is taken of the gauge transformation properties of the bilocal pion fields. Comments are made on the location of the vector dominance mechanism in this treatment.

I. INTRODUCTION

The planned experimental program at the Continuous Electron Beam Accelerator Facility (CEBAF) will subject the electromagnetic (EM) structure of hadrons and nuclei to detailed scrutiny. Effective field theory descriptions of interacting hadrons usually incorporate the intrinsic nonlocality through empirical form factors to simulate the underlying degrees of freedom that are not treated explicitly. The EM current for such a model is complicated by the fact that the form factors can induce additional contributions over and above the canonical current appropriate to point particles. Without a description of the composite hadrons in terms of the constituent charged field degrees of freedom, the EM current is not uniquely defined. Gauge invariance and current conservation as implemented through Ward-Takahashi identities provide only a partial constraint. The pion exchange current between nucleons has received the most attention in this regard. Prescriptions based on minimal substitution into momentum variables, including those of form factors, have been developed [1] to impose such partial constraints.

An ideal perspective on this problem would be provided if the composite hadron fields and their interactions could be modeled in a manageable form in terms of the point fields of QCD. The hadronic EM current could then, in principle, be directly related to the bare quark current. Some progress has been made in recent years towards the hadronization [2,3] of simplified models of QCD based on a four-fermion interaction. However, for complex hadronic systems such as nucleons interacting through pion exchange, these methods are far from yielding a transparent and realistic quark basis for the associated EM current. A much simpler situation where a hadronic EM current can be generated from the quark level is provided by the meson sector produced from bosonization of four-quark interaction models of the Nambu–Jona-Lasinio (NJL) type. [4] With the usual contact form of the interaction, the composite $\bar{q}q$ meson modes at the mean field or tree level are point objects. To conduct a more realistic investigation of the relation between the EM currents at the quark and hadronic level, we employ the finite range generalization known as the Global

Color-symmetry Model (GCM). [5] The feature of the GCM that is of interest here, is that the resulting fields for $\bar{q}q$ mesons have finite extent. The purpose of this paper is to develop the conserved EM current of the extended pions in terms of the quark EM current and to show how gauge invariance at the level of effective localized pion fields is realized in the presence of the intrinsic nonlocalities. It is known that Ward-Takahashi identities are not limited to point particles. [6] Here we develop several explicit forms of a Ward-Takahashi identity for the photon-pion coupling, and also discuss the role of gauge transformation properties of the extended pion fields in maintaining gauge invariance of the action.

The NJL model has been developed into a very efficient and accessible representation of hadron dynamics that is believed to capture the important elements of low energy QCD phenomena. [7] In the GCM model a somewhat more fundamental stance is attempted in that quark color currents interact via a phenomenological gluon two-point function which can be modeled to incorporate confinement and asymptotic freedom aspects of QCD. The desirable features of hidden chiral symmetry and dynamical quark mass generation are present, but only a global color symmetry is implemented. The required bilocal field bosonization methods have been developed, and a thorough analysis of the pseudoscalar octet of Nambu-Goldstone bosons has reproduced many elements of effective chiral meson actions, including anomalous terms and current algebra results. [8] The dynamical self-energy amplitudes and related vertices introduce nonlocalities that provide a natural convergence to the quark loop integrals that govern meson dynamics. A parameterization of the effective gluon propagator has been developed that produces satisfactory results for meson masses, coupling constants and decays. [9] We have recently explored a generalization that retains valence quarks to produce a chiral quark-meson baryon model at the mean field level. [10] This model takes advantage of the fact that chiral symmetry and the axial Ward identity [11] force a single function to describe both the quark scalar self-energy amplitude and the distributed vertex for coupling of quarks to the σ and $\vec{\pi}$ mesons. With an absolutely confining property embodied in the quark self-energy amplitudes, a numerical solution [12] in the absence of the pion gives sensible results for many nucleon properties. This work, with one free parameter,

demonstrates the physical operation and sensible outcome of a nucleon model where the lack of a vacuum quark mass-shell cooperates with a scalar bilocal quark condensate induced by nearby valence quarks to produce a constituent mass-shell. The EM coupling to this nucleon model is under investigation, and will be reported elsewhere.

For the pion charge form factor, the NJL model in mean field theory produces a behavior that is only qualitatively correct [13] over the time-like and space-like regions spanned by data. The formulation of the pion charge form factor within the GCM is of interest in its own right because several dynamical features arise that are beyond the capability of the contact NJL interaction. The Bethe-Salpeter pion amplitudes at the mean field level will not be constants and an extra length scale that contributes to the pion charge radius will enter this way. The quark self-energy will be a dynamical quantity rather than a constant constituent mass, and the associated photon vertex with the dressed quarks will acquire a quark momentum dependence. These aspects are dynamically linked because in the chiral limit the Dirac scalar component of the quark self-energy coincides with the pion Bethe-Salpeter amplitude. Finally, the finite range nature of the GCM can accommodate confinement and spurious threshold effects from the quark loops should be absent.

In Section II the bosonization of the GCM in the presence of an external EM field is outlined and involves path integral techniques that implement the change of field variables from quarks to bilocal bose fields. The gauge invariance of the original action is maintained to produce a gauge invariant EM coupling to the composite pions. Effective local fields to describe pion propagation are defined through the localization procedure of Cahill. [2] This results in a factorized representation for the bilocal fields with a well-defined internal structure factor that becomes the ladder Bethe-Salpeter amplitude on the mass-shell. At this stage the effective local pion fields appear in a nonlocal action in which the EM current and charge form factor involve the hadronic form factor of the pion.

In Section III we analyze the gauge invariance of the description. Ward-Takahashi identities are derived for the photon-pion vertex in terms of the relevant inverse propagators for the bilocal pion fields and also for the effective local pion fields. In the latter case, the role of

the intrinsic hadronic amplitudes for the pion are displayed. The explicit gauge invariance of the action with first-order photon coupling is analyzed in terms of the transformation properties of the extended $\bar{q}q$ pion fields. From this point of view, the nonlocality of the EM coupling to extended pions is shown to generate the longitudinal component of the four-point vertex associated with pion electro-production on dressed quarks. The role and location of the vector dominance mechanism in this approach is discussed briefly in Section IV. A summary is made in Section V.

II. GLOBAL COLOR-SYMMETRY MODEL

A. Bosonization in a Background Electromagnetic Field

We briefly outline the steps necessary to bosonize the partition function of the GCM in the presence of a background EM field through the use of bilocal auxiliary fields. The techniques are a straightforward generalization of those previously applied [5,8] to the GCM in the absence of a background field and we point out the new elements that arise here. Although our treatment deals with two quark flavors, for which the charge operator is $\hat{Q} = \frac{1}{2}(\tau_3 + \frac{1}{3})$, the formalism is easily generalized in that respect. The action for the GCM with a background EM field can be written in Euclidean metric as

$$S[\bar{q}, q, A_\nu] = \int d^4x d^4y \Big\{ \bar{q}(x) \big[(\gamma \cdot \partial_x + m - i\gamma_\nu \hat{Q} A_\nu(x))\delta(x-y) \big] q(y) + \frac{g^2}{2} j_\nu^a(x) D(x-y) j_\nu^a(y) \Big\}, \tag{1}$$

where m is a small current quark mass, A_ν describes an external EM field, and $j_\nu^a(x) = \bar{q}(x)\frac{\lambda^a}{2}\gamma_\nu q(x)$ is the quark current. This action is invariant under the gauge transformation

$$\begin{aligned} q(x) &\to q'(x) = e^{i\hat{Q}\theta(x)} q(x), \\ \bar{q}(x) &\to \bar{q}'(x) = \bar{q}(x) e^{-i\hat{Q}\theta(x)}, \\ A_\nu(x) &\to A'_\nu(x) = A_\nu(x) + \partial_\nu \theta(x). \end{aligned} \tag{2}$$

The phenomenological gluon two-point function $D(x-y)$ is a parameter function of the GCM and an explicit form that fits low energy meson dynamics is available. [9] The details are not important for the present analysis. The standard bosonization technique [8,14] can be applied in the presence of the external EM field to expose bilocal $\bar{q}q$ fields and their electromagnetic interactions. The current-current interaction is Fierz-reordered through the identity

$$\left(\frac{\lambda^a}{2}\gamma_\nu \mathbf{1}_F\right)_{ij}\left(\frac{\lambda^a}{2}\gamma_\nu \mathbf{1}_F\right)_{kl} = \left(\Lambda^\phi\right)_{il}\left(\Lambda^\phi\right)_{kj}, \tag{3}$$

where the quantities Λ^ϕ are given by

$$\Lambda^\phi = \frac{1}{2}\left(\mathbf{1}_D, i\gamma_5, \frac{i}{\sqrt{2}}\gamma_\nu, \frac{i}{\sqrt{2}}\gamma_\nu\gamma_5\right)\otimes\left(\frac{1}{\sqrt{2}}\mathbf{1}_F, \frac{1}{\sqrt{2}}\vec{\tau}\right)\otimes\left(\frac{4}{3}\mathbf{1}_C, \frac{i}{\sqrt{3}}\lambda^a\right), \tag{4}$$

which we write as $\Lambda^\phi = \frac{1}{2}K^a F^b C^c$. The product of currents in (1) becomes

$$j_\nu^a(x) j_\nu^a(y) = -J^\phi(x,y) J^\phi(y,x), \tag{5}$$

where $J^\phi(x,y) = \bar{q}(x)\Lambda^\phi q(y)$ and we use a summation convention for repeated indices. The gauge invariance of the current-current interaction is preserved by Fierz reordering. The flavor sum on the right hand side of (5) splits into multiplets whose members mix under gauge transformations. In general, fields corresponding to a complete multiplet must be retained for manifest gauge invariance. An example is the pair of charged pions and we shall eventually limit our considerations to this case.

Field variables that eventually take up the role of boson modes of the model are introduced through multiplication of the partition function, $Z\left[A_\nu\right] = N\int D\bar{q}Dq e^{-S[\bar{q},q]}$, by unity in the form

$$1 = \mathcal{N}\int D\mathcal{B}^\phi exp\left\{-\int d^4x d^4y \frac{\mathcal{B}^\phi(x,y)\mathcal{B}^\phi(y,x)}{2g^2 D(x-y)}\right\}. \tag{6}$$

With a shift of integration variables $\mathcal{B}^\phi(x,y) \rightarrow \mathcal{B}^\phi(x,y) + g^2 D(x-y) J^\phi(y,x)$, the current-current term $\sum_\phi J^\phi(x,y)J^\phi(y,x)$ is eliminated from the action in favor of a Yukawa coupling term $\bar{q}\mathcal{B}q$ and a quadratic term in $\mathcal{B}$. The partition function is then given by $Z[A_\nu] = N\int D\bar{q}DqD\mathcal{B}e^{-S[\bar{q},q,\mathcal{B}]}$ where the new action is

$$S\left[\bar{q},q,\mathcal{B},A_\nu\right] = \int d^4xd^4y \left\{ \bar{q}(x)\mathcal{G}^{-1}(x,y)q(y) + \frac{\mathcal{B}^\phi(x,y)\mathcal{B}^\phi(y,x)}{2g^2 D(x-y)} \right\}, \tag{7}$$

and the inverse quark propagator is

$$\mathcal{G}^{-1}(x,y) = \left(\gamma\cdot\partial_x + m - i\hat{Q}\gamma_\nu A_\nu(x)\right)\delta(x-y) + \Lambda^\phi \mathcal{B}^\phi(x,y). \tag{8}$$

The boson fields $\mathcal{B}^\phi(x,y)$ have the same EM gauge transformation properties as the bilocal currents $J^\phi(y,x)$, and the bosonized action (7) is gauge invariant. In particular, under a gauge transformation it is evident that

$$\left[\Lambda^\alpha \mathcal{B}^\alpha(x,y)\right]' = e^{i\hat{Q}\theta(x)}\left[\Lambda^\beta \mathcal{B}^\beta(x,y)\right]e^{-i\hat{Q}\theta(y)}. \tag{9}$$

The transformation law can thus be written

$$\mathcal{B}^\alpha(x,y)' = \Omega_{\alpha\beta}(x,y)\mathcal{B}^\beta(x,y), \tag{10}$$

where the transformation matrix $\Omega_{\alpha\beta}$ is given by

$$\Omega_{\alpha\beta}(x,y) = tr_f\left[F^\alpha e^{i\hat{Q}\theta(x)}F^\beta e^{-i\hat{Q}\theta(y)}\right], \tag{11}$$

if the flavor basis has the orthonormal property $tr_f[F^\alpha F^\beta] = \delta_{\alpha\beta}$. These transformation properties reflect the fact that the bilocal fields are constructed from charged constituents. In the local limit where $\mathcal{B}^\alpha(x,y) \rightarrow \delta(x-y)b^\alpha(x)$, uncharged meson fields that couple to quarks via flavor matrices $\mathbf{1}$ and τ_3 become gauge invariant, while the standard gauge transformations operate for local charged fields. For bilocal fields of finite extent, even the uncharged fields will mix and transform under a gauge change. We shall confine our attention to charged pion modes where the gauge transformation properties signal an electromagnetic coupling to both the propagation coordinate $R = (x+y)/2$ and the substructure coordinate $r = x-y$. Later we shall relate this to the fact that the electromagnetic current for the composite pion field is the usual point form times the charge form factor.

Since the quark fields now occur only quadratically in (7), the bosonization is completed by Grassmann integration to give $Z\left[A_\nu\right] = N\int D\mathcal{B}e^{-S\left[\mathcal{B}^\phi,A_\nu\right]}$ where the gauge invariant boson action is

$$S\left[\mathcal{B}^{\phi}, A_{\nu}\right] = -TrLn\mathcal{G}^{-1}\left[\mathcal{B}^{\phi}, A_{\nu}\right] + \int d^4x d^4y \frac{\mathcal{B}^{\phi}(x,y)\mathcal{B}^{\phi}(y,x)}{2g^2 D(x-y)}. \tag{12}$$

The classical field configurations $\mathcal{B}_0^{\phi}$ are defined by $\frac{\delta S}{\delta \mathcal{B}_0^{\phi}} = 0$ and this produces

$$\mathcal{B}_0^{\phi}(x,y) = g^2 D(x-y) tr\left[\Lambda^{\phi}\mathcal{G}_0(x,y)\right], \tag{13}$$

where the associated propagator $\mathcal{G}_0$ depends self-consistently on $\mathcal{B}_0^{\phi}$. In particular,

$$\mathcal{G}_0^{-1}(x,y) = \left(\gamma\cdot\partial_x + m - i\hat{Q}\gamma_{\nu}A_{\nu}(x)\right)\delta(x-y) + \Sigma(x,y), \tag{14}$$

where $\Sigma(x,y) = \Lambda^{\phi}\mathcal{B}_0^{\phi}(x,y)$ represents the quark self-energy to the present level of treatment and satisfies

$$\Sigma(x,y) = \frac{4}{3} g^2 D(x-y)\gamma_{\nu}\mathcal{G}_0(x,y)\gamma_{\nu}. \tag{15}$$

This is obtained from (13) by reversing the Fierz reordering and carrying out the color sum to produce the $\frac{4}{3}$ factor. Because of the background electromagnetic field, the quark propagator and self-energy are not translationally invariant. In the limit $A_{\nu} \to 0$, Σ and $\mathcal{G}_0$ can be chosen to depend only on $x-y$ and (15) becomes the ladder Schwinger-Dyson equation with a bare vertex. We use the notation $G_0(x-y) = \mathcal{G}_0(x,y)|_{A_{\nu}=0}$, and the momentum representation will be denoted by $G_0^{-1}(q) = i\gamma\cdot q A(q^2) + m + B(q^2)$. Only the first-order dependence of $\mathcal{G}_0^{-1}$ upon A_{ν} is of interest and this will generate the EM vertex with dressed quarks.

The action may be expanded about the classical minimizing configuration to obtain

$$S\left[\mathcal{B}^{\phi}, A_{\nu}\right] = S\left[\mathcal{B}_0^{\phi}, A_{\nu}\right] + \hat{S}\left[\hat{\mathcal{B}}^{\phi}, A_{\nu}\right], \tag{16}$$

where $\hat{\mathcal{B}}^{\phi} = \mathcal{B}^{\phi} - \mathcal{B}_0^{\phi}$ are the new field variables for the propagating modes and the corresponding action is

$$\hat{S}\left[\hat{\mathcal{B}}^{\phi}, A_{\nu}\right] = Tr\sum_{n=2}^{\infty}\frac{(-1)^n}{n}\left(\mathcal{G}_0\Lambda^{\phi}\hat{\mathcal{B}}^{\phi}\right)^n + \int d^4x d^4y \frac{\hat{\mathcal{B}}^{\phi}(x,y)\hat{\mathcal{B}}^{\phi}(y,x)}{2g^2 D(x-y)}. \tag{17}$$

Although only color singlet components are present in the classical configurations $\mathcal{B}_0^{\phi}$, the fluctuation fields of the GCM have color singlet and octet components. There are indications that a proper role for the color octet sector is realized through a transformation that

introduces diquark fields for a description of baryons. [2] The electromagnetic current is to be identified from

$$J_\nu(x) = -\left.\frac{\delta S\left[\mathcal{B}^\phi, A_\nu\right]}{\delta A_\nu(x)}\right|_{A_\nu=0} = -\left.\frac{\delta \hat{S}\left[\hat{\mathcal{B}}^\phi, A_\nu\right]}{\delta A_\nu(x)}\right|_{A_\nu=0} . \tag{18}$$

The second equality in (18) expresses the fact that the saddle point action does not contribute to the current. The functional dependence of $S\left[\mathcal{B}_0, A_\nu\right]$ upon A_ν that enters implicitly through $\mathcal{B}_0\left[A_\nu\right]$ makes no contribution since $\frac{\delta S}{\delta \mathcal{B}_0} = 0$. The explicit A_ν dependence that enters through the bare coupling term of $\mathcal{G}_0^{-1}$ produces a current $-tr\left(G_0(x,x)\hat{Q}\gamma_\nu\right)$ which vanishes due to symmetric integration. The saddle point action can contribute to higher order in A_ν beginning with a vacuum polarization insertion for the photon propagator. We here ignore such phenomena since we seek only the current to which a background electromagnetic field couples. The saddle point action at $A_\nu = 0$ is a constant and is absorbed into the normalization of the partition function which now is

$$Z\left[A_\nu\right] = N\int D\hat{\mathcal{B}} e^{-\hat{S}\left[\hat{\mathcal{B}}^\phi\right] + \int d^4x J_\nu(x) A_\nu(x) + \dots}, \tag{19}$$

where J_ν is the current for the bosonized action $\hat{S}[\hat{\mathcal{B}}]$.

Our considerations are restricted to pion modes and we adjust the normalization of the fields so that

$$\Lambda^\phi \hat{\mathcal{B}}^\phi(x,y) = i\gamma_5 \vec{\tau}\cdot\vec{\pi}(x,y) \tag{20}$$

$$= i\gamma_5 f_\alpha^\dagger \pi_\alpha(x,y), \tag{21}$$

where α is a summation index that takes the values $\alpha = (0,+,-)$ corresponding to a spherical isospin basis. This basis is convenient because it diagonalizes the quark charge operator $\hat{Q}$. We choose $f_0 = \tau_3$, $f_+ = (\tau_1 + i\tau_2)/\sqrt{2}$ so that $tr_f(f_\alpha^\dagger f_\beta) = 2\delta_{\alpha\beta}$. The corresponding field components π_α are defined similarly. If the flavor basis is hermitian, the bilocal fields are hermitian, that is, $\left[\mathcal{B}^\phi(x,y)\right]^* = \mathcal{B}^\phi(y,x)$. In terms of the basis we have chosen, the corresponding property is $[\pi_+(r;R)]^* = \pi_-(-r;R)$ and $[\pi_0(r;R)]^* = \pi_0(-r;R)$ where $r = x - y$ and $R = (x+y)/2$. We use these coordinates to describe internal dynamics

and propagation respectively. In the momentum representation that we shall use, (q, P) are conjugate to (r, R), and we have $[\pi_+(q;P)]^* = \pi_-(q;-P)$ and $[\pi_0(q;P)]^* = \pi_-(q;-P)$.

The free action is the quadratic term of (17) in the absence of the EM field. With the above definitions it can be written as

$$\hat{S}_2\left[\pi\right] = \frac{1}{2}\sum_{\alpha}\int d^4(P,q',q)\pi^*_\alpha(q';P)\Delta^{-1}(q',q;P)\pi_\alpha(q;P), \tag{22}$$

where the composite pion inverse propagator is

$$\Delta^{-1}(q',q;P) = \delta(q'-q)tr\left[G_0(q_-)i\gamma_5 f_\alpha G_0(q_+)i\gamma_5 f^\dagger_\alpha\right] + \frac{9}{2}\int\frac{d^4r}{(2\pi)^4}\frac{e^{-i(q'-q)\cdot r}}{g^2D(r)}, \tag{23}$$

with tr denoting a trace over spin, flavor and color. Here $q_+ = q + P/2$ and $q_- = q - P/2$ where P is the total momentum of the meson field and q is the internal momentum associated with the $\bar{q}q$ substructure. The first term of the inverse propagator (23) is a quark loop and the second term is a bare mass.

B. Localization

The bilocal fields represent a combination of internal dynamics and propagation dynamics that is difficult to deal with at the same time. A useful approach is to use the free action to define free meson modes in terms of which the bilocal fields may be expanded. This amounts to a determination of internal form factors for the composite mesons leaving an associated local field degree of freedom that describes propagation only. We are following here the localization procedure developed by Cahill [2]. The natural description of free meson modes is through eigenfunctions defined by

$$\int d^4q\Delta^{-1}(q',q;P)\Gamma_n(q;P) = \lambda_n(P^2)\Gamma_n(q';P), \tag{24}$$

where $\lambda_n(P^2)$ is the eigenvalue for the nth mode. The operator is hermitian and the eigenvalues are real. When the total momentum is such that the eigenvalue vanishes, (24) becomes a free equation of motion, and the corresponding Γ_n are the internal form factors for the freely propagating mesons. The condition $\lambda_n(P^2 = -M_n^2) = 0$ thus defines the mass-shell

and also suggests that $\lambda_n(P^2)$ provides the appropriate basis for a composite generalization of the elementary inverse propagator $P^2 + M_n^2$. This is the sense in which one has achieved a localization of the problem. The eigenfunctions Γ_n form a complete orthogonal set in terms of which the bilocal fields may be expanded as

$$\pi_\alpha(q;P) = \sum_n \Gamma_n(q;P)\pi_{\alpha,n}(P), \tag{25}$$

where $\pi_{\alpha,n}(P)$ are the local effective field variables defined by projection.

We shall truncate the expansion to a single term corresponding to the lowest mass m_π. At $P^2 = -m_\pi^2$ the on-mass-shell equation $\int d^4q\Delta^{-1}(q',q;P)\Gamma(q;P) = 0$ can be shown, with the help of (23), to be the ladder Bethe-Salpeter equation for the pionic $\bar{q}q$ bound state. [5] In the chiral ($m_\pi = 0$) limit this coincides with the ladder Schwinger-Dyson equation for the Dirac scalar component of the quark self-energy, that is, $\Gamma(q;0) \propto B(q^2)$. We choose to normalize the pion-$\bar{q}q$ amplitudes so that they are dimensionless and have the chiral limit $\Gamma(q;0) \to B(q^2)/f_\pi$. The scale factor f_π is determined as described below. For just the ground state modes, the free action can be written

$$\hat{S}_2[\pi] = \frac{1}{2}\sum_\alpha \int d^4P\pi_\alpha^*(P)\Delta^{-1}(P^2)\pi_\alpha(P), \tag{26}$$

where Δ^{-1} is the effective inverse propagator defined by

$$\begin{aligned}\Delta^{-1}(P^2) &= \lambda(P^2)\int d^4q\Gamma^*(q;P)\Gamma(q;P) \\ &= \int d^4(q',q)\Gamma^*(q';P)\Delta^{-1}(q',q;P)\Gamma(q;P).\end{aligned} \tag{27}$$

Since the eigenvalue becomes zero on the mass-shell, we have $\Delta^{-1}(P^2) = (P^2 + m_\pi^2)Z(P^2)$. The scale factor f_π^{-1} present in each vertex $\Gamma(q;P)$ is determined so that Z is unity on the mass-shell and thus the fields $\pi_\alpha(P)$ are physically normalized. To first order in the current quark mass, the PCAC result $f_\pi^2 m_\pi^2 = -m < \bar{q}q >$ is reproduced by such an analysis. [9,8]

The localization procedure does not ignore meson substructure but rather produces a dynamically equivalent formulation in terms of local field variables. The hadronic form factors $\Gamma(q;P)$ then enter into the coupling of the effective local pion fields to other fields.

We use this simple but well-defined structure to explore the role of hadronic form factors in a gauge invariant EM coupling. It should be emphasized that the meson fields at this level are bare or tree-level fields in the sense that quantum dressing effects from the cubic and higher order couplings among bilocal boson fields have not been applied. Nevertheless there is significant dynamical content in these bare fields as evidenced by the ladder Bethe-Salpeter structure of the internal form factors. Such dynamics can provide a realistic modeling of the pion charge form factor. [15] However the dynamical relations between the photon-quark vertex, the quark self-energy dressing and the Bethe-Salpeter amplitudes that implement gauge invariance are not easily identified without a unified development.

C. The Pion Electromagnetic Vertex

The pion electromagnetic current is

$$J_\nu(Q) = - \left. \frac{\delta \hat{S}_2\left[\pi, A_\nu\right]}{\delta A_\nu(Q)} \right|_{A_\nu=0}, \tag{28}$$

where the quadratic pion term of the action (17) is to be used. The electromagnetic field occurs only in the quark propagator $\mathcal{G}_0$, and for the quark-photon vertex that arises, we use the definitions

$$\begin{aligned} \Gamma_\nu(p,k;Q) &= (2\pi)^2 \left. \frac{\delta \mathcal{G}_0^{-1}(p,k)}{\delta A_\nu(Q)} \right|_{A_\nu=0} \\ &= \delta(p-k-Q)\hat{Q}\Gamma_\nu\left(\frac{p+k}{2};Q\right). \end{aligned} \tag{29}$$

It is then straightforward to obtain the current in the form

$$J_\nu(Q) = \frac{1}{(2\pi)^2}\sum_\alpha Q_{-\alpha}\int d^4P\pi^*_\alpha(P+\frac{Q}{2})\Lambda_\nu(P;Q)\pi_\alpha(P-\frac{Q}{2}), \tag{30}$$

where the integration over the internal $\bar{q}q$ degree of freedom has been carried out to form the photon-pion vertex as

$$\Lambda_\nu(P;Q) = \int d^4q\Gamma^*(q+\frac{Q}{4};P+\frac{Q}{2})\Lambda_\nu(P,q;Q)\Gamma(q-\frac{Q}{4};P-\frac{Q}{2}). \tag{31}$$

The quantity $\Lambda_\nu(P,q;Q)$ is the photon vertex for bilocal pion fields and is given by the quark loop expression

$$\Lambda_\nu(P,q;Q) = tr\left\{G_0(q_-)i\gamma_5 G_0(q_+ + \frac{Q}{2})\Gamma_\nu(q_+;Q)G_0(q_+ - \frac{Q}{2})i\gamma_5\right\}, \tag{32}$$

where $q_+ = q + \frac{P}{2}$ and $q_- = q - \frac{P}{2}$. The photon-pion vertex arrived at here at tree level is the impulse approximation with ladder Bethe-Salpeter amplitudes and is illustrated in Fig. 1. The charge factors appearing in (30) are derived from the charge operator according to

$$Q_{-\alpha}\delta_{\alpha\beta} = \frac{1}{2}tr_f\left[f_\alpha \hat{Q} f_\beta^\dagger\right]. \tag{33}$$

Only the terms involving Q_+ and Q_- will survive and these are the charges of the u and d quarks respectively. There are several symmetries obtainable from (32) that are useful. The property $\Lambda_\nu(-P,-q;Q) = -\Lambda_\nu(P,q;Q)$ follows from charge conjugation, and when this is combined with a γ_5 transformation, the property $\Lambda_\nu(P,q;-Q) = \Lambda_\nu(p,q;Q)$ results. The immediate consequence for the photon-pion vertex is $\Lambda_\nu(P;Q) = -\Lambda_\nu(-P;Q) = \Lambda_\nu(P;-Q)$. Thus the $\alpha = 0$ term of the pion current (30) vanishes because the integrand is odd in P. This is the familiar result that the electromagnetic field does not couple in first order to a self-conjugate meson. We obtain this here for composites because the eigenfunction expansion for the internal structure has been truncated at the ground state level. Were internally excited meson modes to be included, transition currents connecting different internal modes would arise.

For the remaining charged fields in (30) we use the standard notation $\pi(P) = \pi_+(P) = \pi_-^*(-P)$. With use of the symmetry property that $\Lambda_\nu(P;Q)$ is odd under reversal of P, the quark and antiquark terms may be combined to express the current in the standard form

$$J_\nu(Q) = \frac{q_{\pi^-}}{(2\pi)^2}\int d^4P\pi^*(P+\frac{Q}{2})\Lambda_\nu(P;Q)\pi(P-\frac{Q}{2}), \tag{34}$$

where $q_{\pi^-} = Q_- - Q_+$ is the π^- charge. The field $\pi(P)$ describes a π^- particle incoming with momentum P or a π^+ particle outgoing with momentum $-P$. The previously mentioned momentum symmetries allow the vertex to to be written $\Lambda_\nu(P;Q) = 2P_\nu F_\pi + 2Q_\nu P\cdot Q H_\pi$

where both F_π and H_π are invariant functions of the three variables $(P^2, Q^2, (P\cdot Q)^2)$. For on-mass-shell initial and final pions, we have $(P+\frac{Q}{2})^2 = (P-\frac{Q}{2})^2 = -m_\pi^2$ or equivalently $P^2 + \frac{Q^2}{4} = -m_\pi^2$ and $P\cdot Q = 0$. In that case the surviving vertex $2P_\nu F_\pi(Q^2)$ is purely transverse and $F_\pi(Q^2)$ is the charge form factor compatible with ladder Bethe-Salpeter pions. The on-mass-shell current is always conserved so long as the vertex is calculated from (31) and (32) in such a way as to preserve the important symmetry that $\Lambda_\nu(P;Q)$ is odd in P and even in Q. In particular this will follow, if the relation (29) between the photon-quark vertex Γ_ν and the quark propagator is maintained.

The point meson limit is obtained by use of the $Q=0$ value of the vertex and it can be verified that on the mass shell $\Lambda_\nu(P;Q=0) = 2P_\nu$ and so $F_\pi(0)=1$. Details are given in the Appendix. Thus on the mass-shell, the composite current contains the point current as a factor, and can be written

$$J_\nu(Q) = j_\nu(Q) F_\pi(Q^2), \tag{35}$$

where the point current in position space is the standard result

$$j_\nu(R) = -iq_{\pi^-}\left[\pi^*(R)\partial_\nu\pi(R) - \pi(R)\partial_\nu\pi^*(R)\right]. \tag{36}$$

III. GAUGE INVARIANCE

The action that has been developed to first-order in the field A_ν and to second-order in the localized pion field $\pi(P)$ is

$$\begin{aligned}\hat{S}_2[\pi, A] = &\int d^4P\,\pi^*(P)\Delta^{-1}(P^2)\pi(P)\\ &- \frac{q_{\pi^-}}{(2\pi)^2}\int d^4(P,Q)\,\pi^*(P+\frac{Q}{2})\Lambda_\nu(P;Q)A_\nu(Q)\pi(P-\frac{Q}{2}).\end{aligned} \tag{37}$$

The explicit gauge invariance can be exhibited through the development of a Ward-Takahashi identity. This should relate the longitudinal vertex to the free inverse propagator so that the gauge change induced in the free action cancels the first order gauge change of the current term.

A. Ward-Takahashi Identities

At the photon-quark level, the vertex defined by (29) satisfies the integral equation

$$\Gamma_\nu(q;Q) = -i\gamma_\nu - \frac{4}{3}g^2 \int \frac{d^4k}{(2\pi)^4} D(q-k)\gamma_\mu G_0\left(k+\frac{Q}{2}\right)\Gamma_\nu(k;Q)G_0(k-\frac{Q}{2})\gamma_\mu. \tag{38}$$

This is easily obtained from the definitions (14) and (15) for the vacuum quark propagator and self-energy respectively. The Ward-Takahashi identity satisfied by this vertex is

$$Q_\nu\Gamma_\nu(q;Q) = G_0^{-1}\left(q-\frac{Q}{2}\right) - G_0^{-1}\left(q+\frac{Q}{2}\right). \tag{39}$$

Since the photon vertex with bilocal pion fields is given by (32) in terms of Γ_ν, a Ward-Takahashi identity for photon-pion coupling is readily obtained in the form

$$Q_\nu\Lambda_\nu(P,q;Q)\delta(q'-q) = \Delta^{-1}\left(q'+\frac{Q}{4},q+\frac{Q}{4};P_+\right) - \Delta^{-1}\left(q'-\frac{Q}{4},q-\frac{Q}{4};P_-\right), \tag{40}$$

where $P_+ = P + \frac{Q}{2}$ and $P_- = P - \frac{Q}{2}$ are pion total momenta and $\Delta^{-1}(q',q;P)$ is the inverse propagator for bilocal pion fields given in (23). For localized pion fields, the relevant vertex is the expectation value of $\Lambda_\nu(P,q;Q)$ with respect to the internal pion form factors according to (31). The resulting Ward-Takahashi identity is

$$\begin{aligned} Q_\nu\Lambda_\nu(P;Q) = &\int d^4(q',q)\Gamma^*(q';P_+) \\ &\times \left[\Delta^{-1}\left(q',q+\frac{Q}{2};P_+\right) - \Delta^{-1}\left(q'-\frac{Q}{2},q;P_-\right)\right]\Gamma(q;P_-). \end{aligned} \tag{41}$$

This result can be brought closer to a form involving the difference of effective inverse propagators for localized fields by recognizing that the form factors in (41) are eigenfunctions of the propagators therein. With use of (24), the identity (41) becomes

$$Q_\nu\Lambda_\nu(P;Q) = \left\{\lambda(P_+^2) - \lambda(P_-^2)\right\} \int d^4q\Gamma^*(q+\frac{Q}{4};P_+)\Gamma(q-\frac{Q}{4};P_-). \tag{42}$$

The right hand side of this Ward-Takahashi identity should be contrasted with the purely local result $\Delta^{-1}(P_+^2) - \Delta^{-1}(P_-^2)$ appropriate to elementary fields. In the present case, the effective inverse propagator Δ^{-1} for the localized factor $\pi(P)$ of the bilocal pion field contains

field substructure information, namely the pion-$\bar{q}q$ vertices as shown in (27). Half of the momentum transfer from the photon is taken up by the internal structure of the pion and the final factor in (42) reflects this. If the Bethe-Salpeter amplitudes $\Gamma(q;P)$ are everywhere replaced by their on-mass-shell limits, the result can be written as

$$Q_\nu \Lambda_\nu(P;Q) = \left\{\Delta^{-1}(P_+^2) - \Delta^{-1}(P_-^2)\right\} \int d^4q \hat{\Gamma}^*(q + \frac{Q}{4})\hat{\Gamma}(q - \frac{Q}{4}). \tag{43}$$

This corresponds to the form expected for a point pion supplemented by a substructure form factor. In (43), $\hat{\Gamma}$ is normalized so that $\int d^4q \hat{\Gamma}^*(q)\hat{\Gamma}(q) = 1$, and the final factor in (43) is a dimensionless probability amplitude for momentum transfer $\frac{Q}{2}$ to be taken up by the pion internal wave function. Further use of the on-mass-shell limit, shows that the difference of inverse propagators in (43) becomes $2P \cdot Q$ and the standard point limit of the vertex at $Q = 0$ is apparent. For $Q \neq 0$, only the longitudinal vertex is accessible this way, and the pion charge form factor cannot be isolated by considerations of gauge invariance alone. It is clear that the on-mass-shell longitudinal vertex contains only the dynamics of the Bethe-Salpeter amplitudes and does not contain the effects of vector-meson dominance or possible threshold effects associated with the quark loop.

B. Gauge Transformation of the Free Action

The explicit gauge invariance of the action (37) can now be demonstrated. Under the infinitesimal change $\delta A_\nu(x) = \partial_\nu \theta(x)$, or equivalently, $\delta A_\nu(Q) = iQ_\nu \theta(Q)$, the change in the $J \cdot A$ term of the action can be related to free propagation quantities by using the Ward-Takahashi identity (42) for $Q_\nu \Lambda_\nu(P;Q)$. If, in turn, the gauge change in the free action is computed from $\delta\pi(R) = iq_{\pi^-}\theta(R)\pi(R)$, or equivalently $\delta\pi(P) = iq_{\pi^-} \int d^4K \theta(P-K)\pi(K)$, there will be no cancellation. The resolution of this problem is that the effective inverse propagator Δ^{-1} has field content that also transforms under a gauge transformation. The bilocal fields have been factorized as $\pi(q;P) = \Gamma(q;P)\pi(P)$ with the internal amplitudes forming part of the inverse propagator for the field $\pi(P)$. The gauge transformation properties of the bilocal pion field are not simply those of $\pi(P)$.

The gauge change induced in the internal factor of the field can be deduced from the relations (10) and (11). For the charged pions, the complex field $\pi(x,y)$ does not mix with any other field under the gauge transformation, and we have

$$\pi(x,y)' = \Omega(x,y)\pi(x,y), \tag{44}$$

where

$$\begin{aligned}\Omega(x,y) &= e^{iQ_-\theta(x)}e^{-iQ_+\theta(y)} \\ &= e^{iq_{\pi^-}\theta(R)}\omega(r;R).\end{aligned} \tag{45}$$

Here Q_+ and Q_- are quark charges, and the second equality has identified the phase factor containing the pion charge $q_{\pi^-} = Q_- - Q_+$ that is appropriate for a local field. The remaining factor $\omega(r;R)$ implements the extra gauge change that occurs because of the nonlocality and is

$$\omega(r;R) = e^{iQ_-[\theta(x)-\theta(R)]}e^{-iQ_+[\theta(y)-\theta(R)]}. \tag{46}$$

The local limit is $\omega(0;R) = 1$. The first order gauge change in the bilocal pion field is

$$\begin{aligned}\delta\pi(r;R) &= \Big\{iQ_-\theta(x) - iQ_+\theta(y)\Big\}\pi(r;R) \\ &= \Big\{iq_{\pi^-}\theta(R) + iQ_-\left[\theta(x)-\theta(R)\right] - iQ_+\left[\theta(y)-\theta(R)\right]\Big\}\pi(r;R).\end{aligned} \tag{47}$$

In a momentum representation, and with the original pion field factorized into the internal $\bar{q}q$ amplitude and the localized field, the first relation from (47) becomes

$$\delta\pi(q;P) = -\frac{i}{(2\pi)^2}\int d^4Q\theta(Q)\left\{Q_+\Gamma(q+\frac{Q}{2};P-Q) - Q_-\Gamma(q-\frac{Q}{2};P-Q)\right\}\pi(P-Q). \tag{48}$$

An alternative expression in which the purely local result is isolated as a separate term can be obtained from (48) through Taylor expansions in the first argument of the amplitudes Γ about the value q. The result is the momentum representation of the second of the relations (47), and is

$$\delta\pi(q;P) = \frac{1}{(2\pi)^2}\int d^4Q\,\{iq_{\pi^-}\theta(Q)\Gamma(q;P-Q) + \delta A_\nu(Q)W_\nu(q;P,Q)\}\,\pi(P-Q). \quad (49)$$

Here the new quantity introduced in the second term is

$$W_\nu(q;P,Q) = -[Q_+ d_\nu(q,Q) + Q_- d_\nu(q,-Q)]\Gamma(q;P-Q), \quad (50)$$

where

$$d_\nu(q,Q) = \left\{\frac{1}{2}\int_0^1 d\eta e^{\eta\frac{Q}{2}\cdot\frac{\partial}{\partial q}}\right\}\frac{\partial}{\partial q_\nu}. \quad (51)$$

All powers of derivatives with respect to the relative $\bar{q}-q$ momentum q have been collected into W_ν together with the accompanying powers of the photon momentum Q except for one factor of Q_ν which is used to identify $\delta A_\nu(Q) = iQ_\nu\theta(Q)$. In position space, these developments are equivalent to Taylor expansions of $\theta(x)$ and $\theta(y)$ about $\theta(R)$ in (47).

The amplitude W_ν enters only because the pion has size in that the form factor $\Gamma(q;P)$ supports derivatives with respect to the internal momentum q. It is a necessary consequence of gauge transformations on the quark-antiquark field content and modifies the internal $\bar{q}q$ vertex of the pion to accommodate coupling to the (longitudinal) A_ν field generated by a gauge change. The corresponding physical process is electro-absorption of a pion on a quark as illustrated in Fig. 2. Only $Q_\nu W_\nu(q;P,Q)$ contributes to the field change (49) and thus only the longitudinal component of the four-point electro-absorption vertex enters the present discussion of gauge invariance. That information is more conveniently represented in (48).

Under an infinitesimal gauge transformation, the free action changes by

$$\delta\hat{S}_2[\pi] = \int d^4P\lambda(P^2)\left\{f(P) + f^*(P)\right\}, \quad (52)$$

where

$$f(P) = \int d^4q\pi^*(q;P)\delta\pi(q;P). \quad (53)$$

Use of (48) for $\delta\pi$ produces

$$f(P) = \frac{iq_{\pi^-}}{(2\pi)^2}\int d^4Q\theta(Q)\pi^*(P)\pi(P-Q)\int d^4q\Gamma^*(q+\frac{Q}{4};P)\Gamma(q-\frac{Q}{4};P-Q). \tag{54}$$

In obtaining this form, the two terms in (48) weighted by quark charges have been combined into a single term weighted by $q_{\pi^-} = Q_- - Q_+$ through a shift of the integration variable q and by use of the property that $\Gamma(q;P)$ is even in q. In combination with the $f^*(P)$ contribution, the change in the free action becomes

$$\delta\hat{S}_2[\pi] = \frac{iq_{\pi^-}}{(2\pi)^2}\int d^4(P,Q)\pi^*(P_+)\theta(Q)V(P;Q)\pi(P_-), \tag{55}$$

where $P_+ = P + \frac{Q}{2}$ and $P_- = P - \frac{Q}{2}$, and

$$V(P;Q) = \left\{\lambda(P_+^2) - \lambda(P_-^2)\right\}\int d^4q\Gamma^*(q+\frac{Q}{4};P_+)\Gamma(q-\frac{Q}{4};P_-). \tag{56}$$

The property that $\Gamma(q;P)$ is even in P (and hence is real) has been used. The quantity $V(P;Q)$ is exactly $Q_\nu\Lambda_\nu(P;Q)$ due to the Ward-Takahashi identity (42). The generated change in the free action can therefore be written

$$\begin{aligned}\delta\hat{S}_2[\pi] &= \frac{q_{\pi^-}}{(2\pi)^2}\int d^4(P,Q)\pi^*(P_+)\Lambda_\nu(P;Q)\delta A_\nu(Q)\pi(P_-)\\ &= \int d^4QJ_\nu(Q)\delta A_\nu(Q),\end{aligned} \tag{57}$$

where we have constructed $\delta A_\nu(Q) = iQ_\nu\theta(Q)$.

Thus the gauge change in the total action $\hat{S}_2[\pi] - J\cdot A$ is zero and gauge invariance to first order is explicit. The implication of this analysis is that if the gauge transformation properties of the internal $\bar{q}q$ structure factor of the bilocal pion field were ignored, then (56) for $V(P;Q)$ would become $\Delta^{-1}(P_+^2) - \Delta^{-1}(P_-^2) \neq Q_\nu\Lambda_\nu(P;Q)$. This becomes the point limit on the mass-shell and gauge invariance for composite pions cannot be maintained. Although the free action for composite pions can be written in the effective local form $\int d^4P\pi^*(P)\Delta^{-1}(P^2)\pi(P)$, it is not possible to gauge this action by minimal substitution and also accommodate the distributed charge form factor for composite pions. It is necessary to recognize the substructure field content of $\Delta^{-1}(P^2)$ and develop the additional response to a gauge change that this introduces.

We note that the equivalence between the two expressions (48) and (49) for the gauge change of the composite pion field implies a Ward-Takahashi identity for the electro-absorption vertex. If we denote the $\bar{q}q$ vertex of the pion in flavor channels $\alpha = (+,-)$ by $\Gamma^\alpha(q;P) = i\gamma_5 f_\alpha^\dagger \Gamma(q;P)$, then the Ward-Takahashi identity satisfied by the corresponding four-point vertex $\Gamma_\nu^\alpha(q;P,Q)$ is obtainable from (48) and (49) in the form

$$Q_\nu \Gamma_\nu^\alpha(q;P,Q) = \hat{Q}\Gamma^\alpha(q - \frac{Q}{2}; P-Q) - \Gamma^\alpha(q + \frac{Q}{2}; P-Q)\hat{Q} - \left[\hat{Q}, \Gamma^\alpha(q;P-Q)\right]. \quad (58)$$

This is one of the manifestations of gauge invariant photon coupling to composite pions.

IV. VECTOR-MESON DOMINANCE

Although we have limited our considerations to electromagnetic coupling to pions, the technique can be extended to treat other meson components of the fluctuation fields $\hat{\mathcal{B}}^\phi$ that appear in the action (17). Of particular importance for electromagnetic couplings are the neutral vector mesons (ρ and ω in the present two flavor model) that implement the successful vector dominance concept. Consider the neutral vector meson intermediate state contributions to the photon-pion vertex. The usual dynamical signature for vector dominance is the emergence of the current-field identity in which the hadronic electromagnetic current is equivalent to a linear combination of neutral vector meson fields. That is, a bilinear combination of the photon and vector mesons should appear in the action. An illustration of this is found in the NJL model where an exact current-field identity holds. [16] However, the lowest order occurrence of vector meson modes in the action $\hat{S}[\hat{\mathcal{B}}^\phi, A_\nu]$ of (17) is quadratic. The formulation we have pursued does not immediately yield a term that is bilinear in A_ν and $\hat{\mathcal{B}}^\phi$. Nevertheless it would be incorrect to simply add a vector dominance mechanism to the photon-pion vertex yielded by the present formalism.

The vector dominance mechanism is already included in the dynamical content of the photon-quark vertex $\Gamma_\nu(q;Q)$ defined by the inhomogeneous integral equation (38). There are intermediate $\bar{q}q$ ladder states of vector character that generate a propagator pole when-

ever the photon momentum Q is such that the homogeneous version of the equation has a solution. An illustration is provided in Fig. 3. In the vicinity of a $\bar{q}q$ resonance we have

$$\Gamma_\nu\left(q;Q\right) \approx \frac{\Omega_\nu(q;Q)}{Q^2+M_V^2}, \tag{59}$$

where $\Omega_\nu(q;Q)$ is the residue at the pole, and satisfies the equation

$$\Omega_\nu\left(q;Q\right) = -\frac{4}{3}g^2 \int \frac{d^4k}{(2\pi)^4} D(q-k)\gamma_\mu G_0\left(k+\frac{Q}{2}\right)\Omega_\nu\left(k;Q\right)G_0\left(k-\frac{Q}{2}\right)\gamma_\mu, \tag{60}$$

for $Q^2 = -M_V^2$. This is the ladder Bethe-Salpeter equation for a vector $\bar{q}q$ bound state. The tree-level isoscalar and isovector mesons are degenerate and the flavor structure is implemented simply by the quark charge operator $\hat{Q}$ which multiplies both Γ_ν and Ω_ν. This is the finite range counterpart of the observation previously made in the context of the contact NJL model where explicit calculation [13] exhibits the ρ meson peak. The resulting pion charge form factor [17] has the correct qualitative behavior with a vector dominance peak generated by the Q dependence of $\Gamma_\nu(q;Q)$.

V. SUMMARY

The composite pion modes of the finite range four-fermion GCM model have been used to study several issues that arise in relation to a hadronic EM current and its basis at the quark level. A Ward-Takahashi identity is obtained in explicit form because the model allows a propagator for the composite pion to be identified. When the dynamics is cast into the form of a nonlocal action for effective local pion fields, the role of the hadronic $\bar{q}q$ form factors in maintaining the gauge invariance of the EM coupling is developed. The effective inverse propagator for the localized fields contains the substructure information and thereby has a gauge transformation property generated by the extended nature of the bilocal field content. This needs to be included along with the standard gauge transformation of effective local pion fields in order for the gauge change in the free pion action to be exactly cancelled by that of the EM current term via the Ward-Takahashi identity.

The modification to the pionic $\bar{q}q$ vertex due to a gauge change can be viewed as the longitudinal component of a pion-quark electro-absortion vertex. The involvment of this modified vertex in the EM coupling to Bethe-Salpeter model pions has been observed before. [15] Here, however, there is no need to supplement the sum of one-body currents of the impulse approximation by an additional term [15] based on this modified vertex to achieve gauge invariance. The sum of one-body currents is automatically gauge invariant here because the necessary dynamical relations among the elements of the triangle diagram are generated from gauge invariance at the bare quark level. Explicit knowledge of such a four-point vertex would be needed to treat electromagnetic coupling to interactions between bilocal $\bar{q}q$ fields rather than just the kinetic term. In particular, the contributions to the EM current from previously derived [5,9] couplings such as $\rho\pi\pi$ and $\omega\rho\pi$ could be addressed with extensions of the present approach. In a linear sigma model format of the GCM where an extended $\bar{q}q$ scalar is included, the same mechanism will generate a photon-sigma-quark vertex. A gauge invariant EM coupling to the chiral quark-meson model baryon [12] of the GCM will not be possible without explicit inclusion of such a mechanism.

The pion charge form factor at mean field level within the GCM includes several physical phenomena that are not addressed by the contact NJL model. The dressed quark-photon vertex $\Gamma_\nu(q;Q)$ and the quark self-energy depend on the loop momentum and a characteristic length scale should be near the pion size. The pion Bethe-Salpeter amplitudes also depend on the loop momentum here and operate on the same length scale. Whether these elements can combine with the vector dominance singularity to produce the pion charge radius remains to be seen. The GCM can be used to introduce a dynamical quark self-energy that is confining through the absence of a pole in the propagator for real p^2. This removes a threshold singularity that can influence the charge radius just as strongly as the vector meson pole.

It has recently been argued [18] that even with confined quarks, the charge form factor at low momenta can be influenced by a threshold singularity related to an effectively free behavior of quarks. This would be driven by the bound state wavefunction character of the pion $\bar{q}q$ amplitudes with a scale set by the effective binding energy of the pion. This

emphasizes the need for field theory models that accomodate pion size.

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NORMALIZATION

The normalization of the pion-photon vertex is such that $\Lambda_\nu(P;Q=0)=2P_\nu$ on the mass shell. We outline a verification of this. From (31), we require

$$\Lambda_\nu(P;0)=\int d^4q\Gamma^*(q;P)\Lambda_\nu(P,q;0)\Gamma(q;P), \tag{61}$$

where $\Lambda_\nu(P,q;0)$, the photon vertex with bilocal pion fields, is seen from (32) to involve the quark-photon vertex $\Gamma_\nu(q_+;0)$. This in turn is determined by the integral equation (38), which in this limit is equivalent to the Ward identity $\Gamma_\nu(q;0)=-\frac{\partial}{\partial q_\nu}G_0^{-1}(q)$. This enables the soft photon limit of the quark loop to be expressed as the momentum derivative of the inverse propagator for the bilocal pions. After use of the symmetry property that $\Lambda_\nu(P;0)$ is odd in P, it is straightforward to obtain

$$\Lambda_\nu(P;0)=\int d^4(q',q)\Gamma^*(q';P)\left[\frac{\partial}{\partial P_\nu}\Delta^{-1}(q',q;P)\right]\Gamma(q;P). \tag{62}$$

The internal momentum integrations produce $\Delta^{-1}(P^2)$, the inverse propagator for the localized pion fields, and we obtain

$$\Lambda_\nu(P;0)=\frac{\partial}{\partial P_\nu}\Delta^{-1}(P^2)-\lambda(P^2)\frac{\partial}{\partial P_\nu}\int d^4q\Gamma^*(q;P)\Gamma(q;P). \tag{63}$$

Here the second term has been simplified through introduction of the eigenvalue λ associated with Γ and Δ^{-1} according to (24). In the on-mass-shell limit $\lambda(-m_\pi^2)=0$, and the first term produces $2P_\nu$ since $\Delta^{-1}(P^2)=(P^2+m_\pi^2)Z(P^2)$.

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FIGURES

FIG. 1. The EM current or vertex for the composite pions of the GCM at tree level. The momenta labels illustrate Eqs.(30), (31) and (32) of the text, which contain both q and $\bar{q}$ contributions. The dynamically dressed quark propagators, the dressed photon-quark vertex and the pion $\bar{q}q$ amplitudes are defined with a consistent ladder structure that maintains gauge invariance.

FIG. 2. The four-point vertex for the electro-absorption of a composite pion on a quark. The longitudinal amplitude $W_\nu(q; P, Q)$ for this process is involved in the EM gauge transformation of the $\bar{q}q$ pion field because of the spatially extended nature. The corresponding four-point vertex has a Ward-Takahashi identity given by Eq.(58).

FIG. 3. (a) The ladder structure of the dressed photon-quark vertex given in Eq.(38). (b) The vector $\bar{q}q$ intermediate state propagator pole that is generated for $Q^2 \approx -M_V^2$.